A Model for the Susceptibility Assessment of Glacial Lake Outburst Floods Based on Physical Processes and the Analytic Hierarchy Process Based on Physical Processes

Jieqing Hou[1, 2*]

1 Institute of Mountain Hazards and Environment, Chinese Academy of Sciences, Chengdu 610041, China
2 School of Engineering Science, University of Chinese Academy of Sciences, Beijing 100049, China

Abstract

Objective: The aim of this study is to develop a susceptibility assessment model for Glacial Lake Outburst Floods (GLOFs) that is based on physical processes and the Analytic Hierarchy Process Based on Physical Processes (AHPBPP), to quantitatively and objectively evaluate the threat of GLOFs, transforming errors from expert experience in traditional models to errors due to the quality of objective input data. Inspired by this, the Law of Relative Quantities with Uncancelled Dimensions is proposed.

Methods: The research introduces the concept of "instantaneous triggering body mechanical energy transfer efficiency" from the perspective of mechanical energy transfer and conversion. Impact indicators are categorized into three major classes and decoupled and valued using mechanical energy for each indicator. An E/F calculation model was constructed, consistent with the concept of the safety factor in reliability theory, and the effectiveness of this model as a susceptibility assessment tool was validated using the AHPBPP method. The study also summarized the AHPBPP, applying it to construct a landslide susceptibility model from model construction, thereby verifying its generalization capability.

Results: The study found that the new model can effectively assess the susceptibility of GLOFs and minimize the influence of subjective experience, providing a new perspective for understanding the dynamic changes of such events. Key indicators include the mechanical energy of the instantaneous triggering body, the mechanical energy of the lake water, and the critical failure Newton's force of the dam body. The analysis indicates that the model has high universality in assessing susceptibility and offers a more comprehensive and systematic framework compared to traditional methods. The AHPBPP demonstrated a certain degree of generalization capability.

Limitations: The potential limitations of the study include difficulties in accurately obtaining some parameters and the need for further validation of the generalization capability of the AHPBPP. Additionally, the accuracy of the model may be constrained by data quality and monitoring technology.

Conclusion: The model proposed in this study has good applicability and universal significance, providing a scientific basis for disaster prevention and mitigation. Compared to existing research, the uniqueness of this work lies in combining quantitative methods based on physical processes with AHPBPP, offering new tools and analytical methods for more accurate identification and assessment of disaster risks. The AHPBPP allows traditional AHP to overcome the limitations of subjective experience.

Keywords: Glacial Lake; Outburst; Analytic Hierarchy Proces; Susceptibility; Quantitative

* Corresponding author: Jieqing Hou (E-mail: houjieqing@imde.ac.cn)

1 Introduction

As sensitive indicators of global change, the cryosphere is not only changing rapidly and significantly but also responding directly and sensitively to the climate system (Qin et al., 2018). Glaciers, as an important component of the cryosphere, are accelerating their melting rate under the backdrop of global warming (Marta, S. et al., 2021; Lee, E. et al., 2021; Carrivick, J. L. et al., 2023). This phenomenon leads to a large amount of glacial meltwater converging, thereby promoting the further expansion of the area of existing glacial lakes, and suitable topographic conditions also contribute to the formation of new unstable glacial lakes, a trend that has persisted in recent years (Nie et al., 2017; Harrison et al., 2018; Shugar, D. H et al., 2020). The latest research data shows that there are currently more than 110,000 glacial lakes globally, covering an area of about 15,000 square kilometers, and the area of these lakes has increased by about 22% from 1990 to 2020 (Zhang et al., 2024). In addition, global warming also promotes the frequent occurrence of extreme climate events, which may further exacerbate the occurrence of factors triggering glacial lake outbursts such as ice/snow avalanches (Richardson et al. , 2000; Nie et al.,2018), debris flows (Richardson et al., 2000; Haeberli et al., 2017), and floods (Emmer, 2013; Emmer, 2017). Glacial lake outburst disasters have become a significant obstacle to the economic and social development of high-altitude and cold regions, historically causing tens of thousands of deaths and massive infrastructure damage (Carey, 2005; Liu, J et al.2014; Allen et al., 2016; Carrivick and Tweed, 2016; Nie et al., 2018). The high-altitude location of glacial lakes means that the floods released during a breach carry huge potential energy and can carry a large amount of material along the way, forming destructive high-sediment floods or debris flows (Cui et al., 2003; Worni et al., 2014; McKillop and Clague, 2007). Currently, more than 10 million people worldwide live under the potential threat of glacial lake outbursts, posing a severe challenge to infrastructure construction and planning in high mountain Asia, constraining the sustainable economic development of the region (Xu, 1988; Clague et al. 2000; Sattar et al. 2022; Nie et al., 2023). For countries where the hydropower industry accounts for a large proportion of national income, the impact of glacial lake outbursts is particularly severe (Carrivick and Tweed, 2016). With the expansion and increase in the number of glacial lakes, and the increasing frequency of human activities within the influence area of glacial lakes, the impact of glacial lake outburst disasters tends to expand, making the analysis of glacial lake outburst susceptibility particularly crucial (Gardelle, J et al.,2011; Carrivick, J. L et al.,2013; Nie et al., 2023). Such analysis helps us to better grasp the dynamic changes of glacial lake threats (Emmer et al. 2018) and provides a scientific basis and guidance for disaster prevention and mitigation.

2 Research Status

The prediction of glacial lake outbursts significantly differs from traditional flood frequency analysis methods in hydrology, primarily because glacial lake outbursts often manifest as one-time events (Steijn, H. V., 1996; Hegglin, E., 2008). Predicting this phenomenon is a complex scientific problem involving high nonlinearity and uncertainty, requiring consideration of the complex dynamic relationships between inputs and outputs, which increases the difficulty of predicting glacial lake outbursts (Zhou, B et al., 2023). Consequently, the academic community has developed various models for assessing the susceptibility to glacial lake outbursts. Based on the composition of assessment models, the

selection of assessment indicators, and the degree of subjectivity in determining the importance of assessment indicators, these models can be broadly classified into qualitative, semi-quantitative, and quantitative categories (Clague, J.J et al., 2000; Wang, X et al., 2007). Among these, qualitative models exhibit the highest subjectivity, semi-quantitative models have moderate subjectivity, and quantitative models have the least subjectivity. Qualitative models overly rely on the experiential knowledge of the evaluator, generalizing assessment indicators based on subjective experience, with threshold settings for assessment indicators also entirely based on subjective experience, resulting in vague and subjective outcomes regarding susceptibility (Carey, M., 2005; Costa, J. E et al., 1988; Huggel, C et al., 2004). Compared to qualitative models, semi-quantitative models quantitatively process assessment indicators based on subjective experience, reducing reliance on the evaluator's experience to some extent due to the application of certain computational methods (Reynolds, J. M, 2003; Bolch, T et al., 2011). Although quantitative models appear more objective in terms of indicator weight distribution and computational processes by utilizing certain tools, they still remain influenced by subjective experience when selecting assessment indicators and computational tools (Mckillop R J et al., 2007; Wang, W et al., 2011; Mergili, M et al., 2011). Furthermore, existing models have not sufficiently considered the coupling effects among different indicators, which may adversely affect the accuracy of prediction results (Zhou, B et al., 2023). The results of these models are often regionally limited, lacking universality, leading to situations where a model is only effective in specific areas (Jiajia Gao et al., 2023), and different evaluators may arrive at varying classification results for the same area, causing confusion for decision-makers. Assessment models based on the Analytic Hierarchy Process (Nitesh Khadka et al., 2021; Zhang, D et al., 2023), fuzzy comprehensive evaluation methods (Wang, W et al., 2011), logistic regression methods (Mckillop R J et al., 2007), and fuzzy matter-element extension methods (Liu, J.F et al., 2012) play crucial roles in specific contexts, but they deviate from the physical processes of glacial lake outbursts and cannot finely reflect the dynamic changes in the susceptibility index of glacial lake outbursts. The susceptibility index of glacial lake outbursts should vary with environmental changes, yet the susceptibility indices derived from existing models do not align with their physical characteristics.

3 Theoretical Basis

3.1 The Process of Glacial Lake Outburst

A deep understanding of the process of glacial lake outbursts is the first step in constructing this model. Glacial lake outbursts occur through the long-term interaction between lake water and moraine dams, catalyzed by climatic factors such as temperature and precipitation (IPCC, 2013; Harrison et al., 2018). They are triggered by certain factors such as earthquakes, ice/snow avalanches, landslides, upstream floods/debris flows, and the melting of ice within the dam (Richardson and Reynolds, 2000; Liu Jing-Jing et al., 2014; Worni et al., 2014). The partial spontaneous/non-spontaneous failure of the dam allows lake water to continuously flow out, generating scouring effects and releasing physical processes of the lake water (Liu, J et al., 2013; Westoby et al., 2014; Robin Neupane et al., 2019). Typically, glacial lake outbursts involve a dynamic cascading process of potential energy and kinetic energy conversion (Somos-Valenzuela et al., 2016; Cui, P et al., 2019). It is noteworthy that the point

of dam failure under wave action is not necessarily the weakest part of the dam. Due to the varying energy levels of the waves and the different resistances of various parts of the dam, the point of failure must be where the energy from the waves (lake water) precisely causes the dam to fail; to achieve an outburst, this failure point must also have the potential to generate sustained overflow within a short period. In reality, the mechanical energy acting at the point of dam failure and the resistance of the dam at that location are what truly influence the glacial lake outburst. Therefore, defining the possible failure locations of the dam is crucial for enhancing the accuracy of the assessment model. Although we currently cannot precisely predict the specific failure location, we can estimate potential failure areas through empirical analysis, thereby improving the accuracy of calculating the susceptibility index of glacial lake outbursts.

3.2 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP), proposed by American operations researcher Thomas L. Saaty in the early 1970s, is a method specifically designed to address complex decision-making problems involving multiple objectives, criteria, factors, and levels (Saaty, T. L., 1980). The general process of this method involves first breaking down the decision-making problem into the objective level, criterion level, and alternative level to form a hierarchical structure model. Following this, pairwise comparisons are made among the elements at the criterion and alternative levels, scoring based on relative importance. Then, consistency checks are conducted on the pairwise comparison matrices to ensure the rationality of the evaluation. Next, weight vectors are calculated through the pairwise comparison matrices, which typically involve the calculation of eigenvalues and eigenvectors. Finally, the weights from the criterion level are combined with those from the alternative level to compute the comprehensive scores of the alternatives and rank them (Saaty, T. L., 1990; Saaty, T. L., 1994; Saaty, T. L., 2008; Saaty, T. L. and Vargas, L. G., 2001; Saaty, T. L., 2005).

The core of the AHP lies in its hierarchical decision-making structure, the assessment and quantification of importance, and the calculation and synthesis of weights. Even if there are variations in computational details or technical methods, as long as these core steps are retained, the method still adheres to the spirit of the AHP. For instance, mathematical tools such as fuzzy logic (Zadeh, L. A., 1965; Dubois, D. & Prade, H., 1980) and the ideal point method (Tzeng, G. H. & Huang, J. J., 2011) can be integrated to assist in weight determination or consistency checks.

The significance of hierarchization is that it allows complex decision-making problems to be logically or standardly divided into different levels, facilitating a more systematic understanding and handling of the issues. Weight synthesis enables decision-makers to consider the impact of multiple criteria on alternatives and determine the relative importance of each criterion. Through synthesis, a comprehensive evaluation result can be derived. This method not only provides a quantitative comparison and a way to select the best alternative but also helps decision-makers make more rational and scientific choices when faced with complex decision-making problems.

Due to its ability to integrate both qualitative and quantitative factors in decision-making, the AHP has been widely applied across various fields. In the context of glacial lake outburst susceptibility assessment, the AHP has also found its application. Wang et al. (2011) developed

a first-order method based on the Analytic Hierarchy Process (AHP) to identify potentially dangerous glacial lakes in the southeastern Tibetan Plateau. Five variables were selected: the area of the parent glacier, the distance between the lake and the glacier terminus, the slope between the lake and the glacier, the average slope of the moraine dam, and the steepness of the glacier terminus. Weights for these variables were assigned using the Fuzzy Consistent Matrix (FCM) method. Subsequently, each variable was classified using statistical threshold values, successfully identifying 8 glacial lakes with extremely high breach risks out of 78 moraine-dammed lakes. Moreover, the method's effectiveness was proven by verifying it against 6 historical lake outbursts. Nitesh Khadka et al. (2021) applied the AHP to assess the susceptibility of glacial lake outbursts in the Mahalangur Himalaya, determining the weights of six key factors such as lake area, expansion rate, distance from the glacier, slope in front of the dam, potential for ice/snow avalanches, and potential for upstream GLOFs through expert pairwise comparisons. The susceptibility index was then calculated, categorizing glacial lakes into different susceptibility levels from very low to very high. Zhang, D et al. (2023) established an assessment method combining the Analytic Hierarchy Process (AHP) with digital elevation models (DEM), glacier data, remote sensing images, and field surveys to quantify the susceptibility to glacial lake outburst floods (GLOFs) in the Nidu Zangbo Basin of the Tibetan Plateau, successfully identifying and validating high-risk glacial lakes.

3.3 Safety Factor

In constructing the analytical model for the susceptibility index of a single glacial lake outburst, the relevant concepts from reliability theory regarding safety factors align well with our approach. The application of reliability theory in assessing the safety of existing structures has been widely recognized (Diamantidis, D et al., 2024). When exploring the safety of existing structures, reliability theory provides a powerful analytical framework, particularly through the introduction of the core concept of the safety factor. The definition of the safety factor (SF) may vary slightly across different engineering fields and standards, but the core concept remains similar. The safety factor is typically defined as the ratio of the load-carrying capacity considered during the design of a structure or component to its actual load requirements under the most unfavorable load combinations (ISO 13822:2017; IBC2018). This factor is used to ensure that structures can safely withstand various anticipated and unexpected loads throughout their design life. In traditional engineering practice, the safety factor is often defined as the ratio of existing strength parameters to the strength parameters required for stability (Song, E.X et al., 2016). Glacial lakes, as products of nature, consist of components such as instantaneous triggering bodies, the glacial lake itself, and the dam. During the process of a glacial lake outburst, these components work together to form a natural existing structural system. The concept of safety factors should also apply in this analysis.

4 Model Construction

4.1 Analysis and Construction of the Indicator System

From a practical standpoint, achieving absolute accuracy in obtaining the susceptibility index of glacial lake outbursts is unrealistic. Our goal is to get as close as possible to the susceptibility index of glacial lake outbursts. To achieve this, an accurate understanding of

random factors and other influencing factors is required. However, due to limitations in technical means, access to information resources, levels of theoretical knowledge, and natural environmental conditions, human understanding of all factors affecting glacial lake outbursts is limited. Particularly for complex influencing factors, the limitations in technical understanding and uncertainties in subjective understanding are particularly evident. This paper aims to improve our understanding of the susceptibility index of glacial lake outbursts under existing conditions.

Therefore, we start from the perspective of mechanical energy transfer and conversion, dividing the three main participants directly involved in the process: instantaneous triggering bodies, the glacial lake itself, and the dam body. The instantaneous triggering bodies include ice/snow avalanches, landslides, debris flows, and floods, while the glacial lake itself includes two parts: the lake water and the lake basin. All indicators affecting glacial lake outbursts act directly or indirectly on these three main bodies, thereby participating in glacial lake outbursts and influencing the susceptibility index of glacial lake outbursts.

For the instantaneous triggering bodies, indicators affecting their susceptibility index of glacial lake outbursts can be divided into two parts: one is the indicators affecting their probability of occurrence, and the other is the indicators affecting the calculation of their mechanical energy entering the lake. Indicators affecting their probability of occurrence include air temperature, precipitation, earthquakes, and their own innate endowments. Indicators affecting the calculation of mechanical energy entering the lake include the mass, shape, density, volume, distance from the lake, height above the lake, and the friction coefficient of the path into the lake of the instantaneous triggering bodies. For the glacial lake itself, indicators related to the susceptibility index of glacial lake outbursts are mainly divided into two parts: indicators related to the lake basin and indicators related to the lake water. Indicators related to the lake basin include the shape, slope, and volume of the lake basin; indicators related to the lake water include the volume, density, and distribution of the lake water. The resistance of the dam body is also crucial, and indicators affecting the susceptibility index of glacial lake outbursts are divided into two parts: indicators related to the shape of the dam body and indicators related to the internal composition of the dam body. Indicators related to the shape of the dam body include the height, width, and upstream slope of the dam body; indicators related to the internal composition of the dam body include the grain size distribution, mineral composition, structural distribution of the dam soil, and the content of ice in the dam body. Indicators related to instantaneous triggering bodies and the glacial lake itself are mainly used for energy and probability calculations, while these dam-related indicators are used for resistance calculations.

In previous assessments of the susceptibility of glacial lake outbursts, the above indicators have often not been fully considered as assessment indicators. However, as physical properties of the main bodies involved in glacial lake outbursts, these indicators cannot be ignored in any assessment model. For example, indicators commonly used for glacial lake outburst susceptibility assessments, such as lake water area, the rate of change of lake water area, the ratio of lake water depth to dam height, can all be seen as representations of lake water, ultimately reflecting the state of mechanical energy of the lake water at the dam failure location; similarly, commonly used indicators such as the area, thickness, distance from the lake, and fracture development of the rear edge glacier can all be seen as representations of instantaneous

triggering bodies, ultimately reflecting the calculation of mechanical energy entering the lake and the probability of occurrence of instantaneous triggering bodies. Furthermore, from the perspective of mechanical energy transfer and conversion, we have identified some new indicators, such as the distance of the instantaneous triggering body relative to the dam failure location, the angle of entry of the instantaneous triggering body into the lake, the wind speed during the movement of the instantaneous triggering body, and the number of blocks disintegrated when the instantaneous triggering body enters the lake, which help to more accurately calculate the mechanical energy of the instantaneous triggering body entering the lake.

4.2 Energy Transfer Efficiency Construction

Since the instantaneous triggering body is generally not in direct contact with the dam failure location, the mechanical energy of the instantaneous triggering body acts on the dam failure location through the glacial lake water in the form of a surge. We consider the surge energy acting on the dam failure location as the energy produced by the instantaneous triggering body that contributes to the glacial lake outburst. Therefore, the ratio of the mechanical energy of the lake water at the dam failure location caused by the instantaneous triggering body to the mechanical energy of the lake water by the instantaneous triggering body is defined as the energy transfer efficiency.

$$\eta = \frac{E_b}{E_h} \tag{1}$$

In the formula, η is the energy transfer efficiency, E_b is the mechanical energy of the lake water at the dam failure location caused by the instantaneous triggering body, and E_h is the mechanical energy of the lake water by the instantaneous triggering body. Since ηη varies with time, η is a function of time t, denoted as $\eta(t)$, with η_t being the moment when the dam failure location receives the energy transmitted by the instantaneous triggering body, and the unit is seconds.

4.3 Construction of Dam Resistance

Glacial lakes are categorized based on their dam types into ice dams, moraine dams, and rock dams (Otto, J.C, 2019). During the process of glacial lake outbursts, the dam body may be subjected to various forces, which can be broadly divided into surface forces and volume forces. Surface forces include buoyancy, pressure from lake water on the dam, etc.; volume forces include seepage forces, gravity, etc. The modes of failure also exhibit diversity, potentially including overflow scouring failure, piping seepage failure, tensile fracture failure, impact shear failure, etc.

The complexity of dam failure stems from multiple factors, including the diversity of failure modes, the variety of forces acting, the complexity of material properties, the irregularity of the dam's geometric shape, and the randomness of the interaction between the surge and the dam, etc. These factors collectively determine that the critical failure Newton's force required for dam failure cannot be precisely calculated, meaning that the resistance of a moraine dam is unsolvable. However, despite the inability to obtain an exact value, resistance still has expressibility.

Numerous factors affect the unsolvability of dam resistance, which can be mainly summarized as the performance of the materials composing the dam, the geometric parameters of the dam, and the calculation model. Material performance involves its strength, elastic modulus, Poisson's ratio, and other physical properties. Due to differences in the composition and structure of dam materials, as well as the influence of environmental conditions, the performance of dam materials may vary. The geometric parameters of the dam, such as height, width, length, slope, etc., may also change under the influence of the environment. These changes may have a minor impact on dam resistance in the short term and are usually treated as constant values in calculations. The calculation of dam resistance is usually based on some basic assumptions, which may not fully match the actual situation, or the calculation formulas themselves may involve approximations, thus introducing variability. Despite this, what we can be certain of is that there exists a specific critical Newton's force at the time of dam failure. This force can be expressed as the product of stress and area, where stress represents material performance, and area represents the geometric parameters of the ice dam. Through the critical failure Newton's force, we can characterize the overall resistance of the dam at the failure location.

The critical failure Newton's force is the combined force of surface and volume forces acting on the material when it is in a critical failure state. There exists a critical failure Newton's force for any failure mode of the dam, whether it is overflow scouring, piping seepage, impact shear, or the coupling of various failure modes.

$$F=\sqrt{F_1^2+F_2^2-2F_1F_2\cos\theta}=\delta cs \qquad (2)$$

In the formula, δ represents the ratio of the actual dam resistance to the calculated resistance; c denotes the shear strength of the dam soil, which is used to characterize the performance of the dam material; s denotes the shear area of the dam, which is used to characterize the geometric parameters of the dam; F denotes the critical failure Newton's force of the dam, which is used to represent the dam's resistance; F_1 denotes the resultant force of the surface forces; F_2 denotes the resultant force of the volume forces; θ denotes the angle between the resultant surface force and the resultant volume force.

From the formula, it can be seen that the critical failure Newton's force is influenced by the combined action of the surface forces and the volume forces at the critical failure state. Any change in the magnitude or direction of a surface force or a volume force will cause the critical failure Newton's force to change. In practical work, we can only obtain soil samples in front of the instantaneous triggering body for analysis, as once the instantaneous triggering body enters the lake, the mechanical energy begins to transfer, and the dam may fail at any time. When the dam failure location receives energy transmitted through the lake water from the instantaneous triggering body, the properties of the soil at the failure location will change, and consequently, the critical failure Newton's force will also change. At this time, the critical failure Newton's force varies with time, denoted as $F(t)$, which represents the critical failure Newton's force as a function of time t, starting from the moment the dam failure location receives energy transmitted through the lake water from the instantaneous triggering body, with the unit being seconds.

4.4 Selection, Decoupling, and Valuation of Indicators

The paper has analyzed and constructed important parameters of the indicator system involved in glacial lake outbursts, which can be summarized into three major indicators: indicators affecting the mechanical energy contributed by the instantaneous triggering body to the dam failure location, indicators affecting the mechanical energy contributed by the lake water to the dam failure location, and indicators affecting the resistance at the dam failure location. As mentioned in the research status, existing models do not fully decouple the interactions between indicators, and the selection and weighting of assessment indicators are still influenced by subjective experience. This section addresses these issues.

Since the glacial lake outburst is a dynamic cascading process involving the transfer and conversion of mechanical energy, the transfer and conversion of mechanical energy run through the entire process of glacial lake outbursts. Therefore, when constructing the model, we took mechanical energy into account, which serves two purposes: one is to value the indicators, and the other is to decouple the interactions between various indicators. We found that when calculating the mechanical energy contributed by the instantaneous triggering body to the dam failure location and the mechanical energy contributed by the lake water to the dam failure location, they are independent and do not interfere with each other. The result reflected in the model is that the weight of different indicators is the amount of mechanical energy they contribute, that is, the larger the value, the greater the weight. In this way, the natural valuation of the two major indicators, the mechanical energy contributed by the instantaneous triggering body to the dam failure location and the mechanical energy contributed by the lake water to the dam failure location, is achieved, and the coupling and valuation issues between indicators are resolved. This paper constructs the model from the perspective of mechanical energy transfer and transformation, and the selection of objective physical process indicators includes all influencing factors without any subjective experience. The resistance at the dam failure location does not belong to the link of mechanical energy transfer and conversion, so it naturally does not couple with other types of indicators. However, the resistance at the dam failure location is also an important indicator affecting the glacial lake outburst, and its value also affects the solution and analysis of the glacial lake outburst susceptibility index. Valuation is also very necessary, and here we use the critical failure Newton's force to value the resistance at the dam failure location.

4.5 Determination of Failure Core and Location

Landslides have a sliding surface, and dam breaches have a failure core. Just as the position of the sliding surface needs to be determined when calculating the stability index of a landslide, the position of the failure core also needs to be determined when calculating the susceptibility index of a glacial lake outburst. Due to the anisotropy of dam materials, that is, the critical failure Newton's force is different everywhere, and the energy received by various parts of the dam is different. When the critical failure Newton's force of a part of the dam cannot resist the water energy, that part of the dam will be destroyed. If this part has the ability to continue to produce overflow after destruction, then the glacial lake has breached. In a dam, there may be multiple places where the dam is destroyed after a surge, but the failure core is the most likely place for the dam to be destroyed, that is, where the ratio of the energy acting on that place and the critical failure Newton's force of that place is the largest. According to

experience, the failure core is more likely to appear at lower positions on the dam surface, where piping is easy to occur, slightly higher than the water level of the lake, and where the mechanical properties of the soil are poor.

5 Model Proposal

After an in-depth analysis of the physical processes and theoretical foundations of glacial lake outbursts, we have categorized the three main participants in the glacial lake outburst system into two primary categories: the attacking party and the defending party. The attacking party includes the instantaneous triggering bodies and the glacial lake itself, which contribute to the mechanical energy at the dam failure location through the instantaneous triggering bodies and the lake water at the dam failure location; the defending party refers to the dam itself, whose defensive capability is represented by the critical failure Newton's force at the dam failure location. Consequently, we have constructed a model for the solution and analysis of the susceptibility index of glacial lake outbursts. This model aims to comprehensively consider various factors within the glacial lake outburst system and to quantitatively solve and analyze the susceptibility index of glacial lake outbursts.

$$(GLOFSI_{(t)})_{\max} = \left(\frac{\sum_{i=j=k=1}^{n} \psi_i \eta_{(t)j} E_{hk} + G_b}{F_{(t)} * 1m}\right)_{\max} = \left(\frac{\sum_{i=j=k=1}^{n} \psi_i \eta_{(t)j} E_{hk}}{F_{(t)} * 1m} + \frac{G_b}{F_{(t)} * 1m}\right)_{\max} \tag{3}$$

$$E_h = E_{all} - E_s \tag{4}$$

At the moment of time t, when the susceptibility index of glacial lake outburst flood (GLOFSI) takes its maximum value, we have:

$$GLOFSI = \frac{\sum_{i=j=k=1}^{n} \psi_i \eta_{tj} E_{hk} + G_b}{F_t * 1m} = \frac{\sum_{i=j=k=1}^{n} \psi_i \eta_{tj} E_{hk}}{F_t * 1m} + \frac{G_b}{F_t * 1m} \tag{5}$$

In the formula, *GLOFSI* (Glacial Lake Outburst Flood Susceptibility Index) is the susceptibility index of glacial lake outburst floods. We are only interested in the specific numerical value of this ratio, so we divide it by 1m to eliminate the units and obtain a dimensionless number; ψ represents the probability of the occurrence of the instantaneous triggering body, with a maximum value of 1 and a minimum value of 0; η_t represents the energy transfer efficiency at time t; E_h represents the mechanical energy of the instantaneous triggering body to the lake water; G_b represents the potential energy of the lake water at the dam failure location; E_{all} represents the initial potential energy of the instantaneous triggering body; E_s represents the mechanical energy lost by the instantaneous triggering body before entering the lake; F_t represents the critical failure Newton's force at the dam failure location at time t, which is used to characterize the overall resistance of the dam; here, the unit of energy is joules, the unit of force is newtons, and the unit of time is seconds.

If we directly use the critical failure energy at the selected location in the denominator, we can directly obtain a dimensionless number. However, we have chosen to represent the dam's resistance with the critical failure Newton's force for the following two main reasons:

Firstly, the critical failure Newton's force is more easily obtainable through technical means compared to the critical failure energy; both *c* (cohesion) and *s* (shear area) can be measured, with only δ (a coefficient of experience) remaining as an empirical factor. In contrast, estimating the critical failure energy involves too many uncertain parameters, leading to significant estimation errors. Secondly, just like the critical failure energy, the critical failure Newton's force can strictly represent the dam's resistance, and regardless of the circumstances, both the critical failure Newton's force and the critical failure energy exist. Thirdly, based on the understanding of the breach phenomenon in this paper, due to the diversity of dam failure modes and the current lack of a clear definition for the breach phenomenon, it is still a matter of debate to determine the extent of dam failure that constitutes a glacial lake outburst. Therefore, this model defines a glacial lake outburst as the dam's ability to quickly generate sustained overflow after a certain volume of destruction, at which point any failure mode has a significant failure location. When overflow failure occurs, the significant failure location has a relatively small volume of destruction; when wave-induced breach occurs, the significant failure location has a relatively large volume of destruction. When piping failure occurs, the piping failure is a process of fine particle migration, which has a certain timescale, thus requiring the identification of the volume at the piping failure location. We define the critical Newton's force that causes the estimated failure volume at the significant failure location to fail as the dam's critical failure Newton's force. We define the volume at the significant failure location as the failure core. Based on the above understanding, this model assumes that the failure location can be infinitely large or infinitely small. For the above considerations, this paper chooses to represent the dam's resistance with the critical failure Newton's force. However, if other forms (including critical failure energy) are used to represent the dam's resistance, a similar *E/F* calculation mode constructed here would still be a valid calculation mode.

6 Model Analysis

When conducting a quantitative analysis of the susceptibility index of a single glacial lake outburst, we find that all influencing factors can be directly or indirectly reflected in the corresponding susceptibility expressions of the participating entities, and they collectively determine the changes in the glacial lake outburst susceptibility index. Importantly, the changes in the glacial lake outburst susceptibility index do not always correspond to dramatic changes in a single parameter; instead, a comprehensive assessment from a systemic perspective is required. For instance, if the cracks in a dangerous ice avalanche body at the rear edge of a glacier have increased, raising the probability of its occurrence, but at the same time, the lake water has significantly decreased due to evaporation and seepage, we cannot solely conclude that the glacial lake outburst susceptibility index has increased just because the probability of ice avalanche occurrence has increased, nor can we conclude that the glacial lake outburst susceptibility index has decreased just because the lake water has decreased. It is necessary to consider the changes in the remaining parameters in the expression. At the same time, changes in factors that we subjectively believe to be influential may not necessarily affect the glacial lake outburst susceptibility index, as these changes must result in changes in the parameters within the expression to influence the glacial lake outburst susceptibility index. This systemic perspective provides a more comprehensive understanding of the changes in the glacial lake

outburst susceptibility index of a single lake.

Each glacial lake has its unique natural characteristics and conditions, and therefore, the susceptibility index expression for each glacial lake is established based on its specific situation. The primary step in analyzing the susceptibility index of a single glacial lake outburst is to identify and understand the natural endowment of the glacial lake, including its geographical location, morphology, hydrological conditions, etc., and to determine the relevant parameters and their values as accurately as possible. Based on these parameters, we can construct a susceptibility index expression specific to that glacial lake and perform the analysis. By analyzing the susceptibility index expression in the form of division, we can observe that within a certain period, if the numerator (representing the energy of the attacking party) decreases while the denominator (representing the resistance of the defending party) increases, the glacial lake outburst susceptibility index will decrease; conversely, if the numerator increases while the denominator decreases, the glacial lake outburst susceptibility index will rise. When both the numerator and the denominator increase or decrease simultaneously, the trend in the glacial lake outburst susceptibility index may not be so apparent. In this case, if we can obtain the specific amplitude of changes in the numerator and the denominator through monitoring or other means, we can more accurately determine the changes in the glacial lake outburst susceptibility index. This is analyzed using the concept of ratio.

The glacial lake outburst susceptibility index expression is also composed of addition, where the numerator is broken down into two parts and each is divided by the denominator to obtain two parts that add up, each representing a different concept. The first part represents the glacial lake outburst susceptibility index contributed by the instantaneous triggering body, and the second part represents the glacial lake outburst susceptibility index contributed by the potential energy of the lake water at the dam failure location. That is to say, the glacial lake outburst susceptibility index consists of two parts: one contributed by the instantaneous triggering body and the other contributed by the lake water. When analyzing the susceptibility index of a single glacial lake outburst, if both parts of the glacial lake outburst susceptibility index increase, the glacial lake outburst susceptibility index also increases; if both parts decrease, the glacial lake outburst susceptibility index also decreases. When one part of the glacial lake outburst susceptibility index increases and the other decreases, we need to solve for the corresponding part of the glacial lake outburst susceptibility index, but current technical means cannot achieve precise valuation. At this time, we can conduct quantitative analysis to determine which part plays a dominant role in the susceptibility index, as the changes in the dominant part often dictate the changes in the glacial lake outburst susceptibility index, especially when the change in the dominant part is greater than that in the non-dominant part. In some cases, although there may be errors in judging the changes in the glacial lake outburst susceptibility index, when the changes in the dominant and non-dominant parts are not easily distinguishable, that is, when the change in the dominant part is small and the change in the non-dominant part is large, the combined changes in the glacial lake outburst susceptibility index will not be significant.

Specifically, for moraine lakes without instantaneous triggering bodies, we can analyze their glacial lake outburst susceptibility index expression and the changes in the susceptibility index under specific environmental changes to determine the changes in the glacial lake outburst susceptibility index.

$$GLOFSI = \frac{G_b}{F * Im} \tag{5}$$

When $\frac{\Delta G_b * F}{\Delta F * G_b} = 1$, The susceptibility index of Glacial Lake Outburst Floods remains unchanged ; (6)

When $\frac{\Delta G_b * F}{\Delta F * G_b} > 1$, the susceptibility index of Glacial Lake Outburst Floods increases; (7)

When $\frac{\Delta G_b * F}{\Delta F * G_b} < 1$, the susceptibility index of Glacial Lake Outburst Floods decreases. (8)

For instance, under conditions of rising temperatures, the meltwater from glaciers increases, but the changes in precipitation and evaporation seepage may be uncertain. Concurrently, the critical failure Newton's force at the dam failure location may decrease due to the melting of buried ice and ice lenses. In such circumstances, we would typically expect the susceptibility index of glacial lake outbursts to increase. However, according to our model of glacial lake outburst susceptibility, a more precise assessment of the changes in the volume of glacial lake water is required, which can be achieved by monitoring the water level corresponding to the assumed breach range of the dam. We also need to evaluate whether the resistance at the dam failure location has decreased or remained almost unchanged. If the water level at the dam failure location rises, we can essentially determine that the susceptibility index of the glacial lake outburst has increased. If evaporation and seepage are too strong, causing the water level at the dam failure location to drop, then the specific changes in the glacial lake susceptibility index require further analysis. But it can be preliminarily judged that, in this case, the increase in the susceptibility index of glacial lake outbursts may not be significant, and it may even decrease.

For glacial lakes with only a single dangerous ice avalanche body, it is crucial to establish an exclusive expression for the glacial lake outburst susceptibility index. With the continuous impact of global warming, the probability of instantaneous triggering bodies occurring may increase. However, as the ice body continues to melt, its volume and potential energy continuously decrease, with high potential energy gradually transforming into the low potential energy of lake water. During this process of mechanical energy transformation, the parameters involved in the calculation of the glacial lake outburst susceptibility index are also constantly adjusting and changing. It is worth noting that during this process, there may be a phenomenon where the glacial lake outburst susceptibility index actually decreases. For example, in the past, ice avalanche bodies entering the lake could trigger outbursts, but when the dangerous ice body melts to the point where the waves caused by its fall into the lake are not sufficient to cause an outburst, this phenomenon is empirically observed. This is contrary to intuitive expectations. At the same time, as glaciers gradually melt, the distance between the rear edge of the glacier and the glacial lake becomes increasingly distant, leading to greater mechanical energy consumption of the dangerous ice avalanche body entering the lake and a decrease in the mass of the dangerous ice avalanche body. Meanwhile, glacial meltwater does not increase the

volume of the glacial lake water due to infiltration and evaporation. At this point, the decrease in the numerator is greater than the decrease in the denominator, which is also evidence of a decrease in the susceptibility index of glacial lake outbursts under global warming for some glacial lakes.

Previous studies have also considered the issue of the minimum glacial lake area threshold (Nie et al., 2018; Veh et al., 2019). Glacial lakes below a certain area threshold are inevitably not considered by researchers and are usually ignored when interpreting glacial lakes through remote sensing images. However, small area glacial lakes can also potentially cause "small breaches leading to large disasters" under the right conditions (Liu, M et al., 2020; Zhang, T et al., 2022). By establishing an expression for the glacial lake outburst susceptibility index, it is not difficult to discover that the overlooked small area glacial lakes may also have a higher susceptibility index than large area glacial lakes, a phenomenon that is not captured in some assessment models that overly rely on area and scale as indicators. This provides a clear signal to glacial lake outburst disaster prevention personnel that although small area glacial lake breaches may not be as severe as large area glacial lake breaches, when the susceptibility index of small area glacial lakes is higher compared to large area glacial lakes, small area glacial lakes should also be given due attention. At the same time, we can also find that no matter how small the area of the glacial lake, it has its own susceptibility index expression. That is, the regional glacial lake outburst susceptibility assessment based on this model does not need to consider the issue of the minimum glacial lake area threshold, further strengthening the universality of this model.

Glacial lakes at lower water levels generally have lower susceptibility indices compared to those at higher water levels, which aligns with our subjective understanding and is also reflected in the glacial lake outburst susceptibility index model we have constructed. When at lower water levels, without considering the changes in instantaneous triggering bodies, the mechanical energy contributed by the lake water is relatively small, and the breach location often shifts downward accordingly. Generally, the lower the dam, the thicker it is, and the greater the Newton's force required for failure. According to the established glacial lake outburst susceptibility index expression analysis, if the numerator decreases and the denominator increases, the result decreases, meaning the glacial lake outburst susceptibility index becomes smaller. At the same time, the possibility of wave-induced breach is smaller because waves need to breach thicker sections to produce sustained overflow. Therefore, in this case, the breach mode of glacial lake outburst is mainly dominated by seepage/piping. Similarly, without considering the changes in instantaneous triggering bodies, as the water level rises, the glacial lake outburst susceptibility index tends to increase with it. When the energy of the waves can easily breach the dam, the breach mode of glacial lake outburst should be wave-induced breach, seepage/piping dominated. As the water level continues to rise, the glacial lake outburst susceptibility index continues to increase, and overflow scouring becomes one of the new dominant breach modes, at which point the breach modes of glacial lake outburst should be overflow scouring, wave-induced breach, seepage/piping dominated.

Through the previous analysis, we can also understand that changes in water levels can cause changes in the susceptibility index of glacial lake outbursts and can even determine the breach mode, indicating that water levels play a controlling role in glacial lake outbursts. When

the water level approaches zero, the entry of instantaneous triggering bodies (except for floods/debris flows) into the lake will not cause glacial lake outbursts; and when the water level reaches its maximum, glacial lakes can easily experience outbursts. Between these two extreme water levels, there must be a critical water level at which the entry of instantaneous triggering bodies into the lake will not trigger an outburst, and this water level is referred to as the safe water level for glacial lakes. Similarly, when the water level of glacial lakes remains stable within a certain range for many years, there is also a suitable instantaneous triggering body that will not trigger an outburst when it enters the lake. The instantaneous triggering body in this case is referred to as the safe instantaneous triggering body. These analyses provide new insights for engineering management: in addition to improving the resistance of the dam, it is also possible to effectively regulate the susceptibility index of individual glacial lake outbursts by lowering the water level to the safe water level, reinforcing the instantaneous triggering body, reducing the mass of the instantaneous triggering body, or simultaneously controlling the water level, instantaneous triggering body, and dam. This control is essentially adjusting the parameter values of the influencing factors to reduce the susceptibility index of glacial lake outbursts, thereby achieving the ideal engineering management effect for glacial lakes, which aligns with our subjective understanding.

This model is entirely constructed based on objective physical processes. Although some parameters may be difficult to obtain precisely due to objective reasons, in certain cases, the model can accurately determine the trend of changes in the susceptibility index of individual glacial lake outbursts. If this model is applied to multiple glacial lakes within a region and the results obtained are ranked, it is possible to conduct a regional susceptibility assessment of glacial lake outbursts, thereby constructing a glacial lake outburst susceptibility assessment model. For conclusions drawn from other evaluation methods, this model can serve as a verification tool under certain conditions.

7 Model Explanation

Since this model is constructed based on physical processes, its architectural form coincides with that of the safety factor, which can lead to confusion with stability analysis. Therefore, it is necessary to clarify the differences between this model and stability analysis models, as well as the similarities between this model and Analytic Hierarchy Process (AHP) models, to illustrate that this model is a susceptibility analysis model rather than a stability analysis model.

In the previous section, we divided the three main entities into the attacking and defending parties, both of which exist within the same glacial lake outburst system. Therefore, the attacking and defending parties hold equal importance but offset each other. In basic arithmetic operations, it is clearly inappropriate to add or multiply the quantities representing the two parties. This leaves us with subtraction and division as the remaining options. When the quantity representing the attacking party is subtracted from that of the defending party, we find that the susceptibility index of large-scale glacial lakes is generally higher than that of small-scale glacial lakes, which is clearly contrary to common sense. This leaves us with division as the only viable option. Thus, we construct the model by dividing the quantities

representing the two parties and find that it meets our initial expectations. In this way, the architectural form of dividing quantities is similar to the safety factor form, but it is different from the safety factor form. The safety factor is usually a dimensionless quantity, while the E/F calculation mode used in this architecture results in a quantity with units. Since we are only interested in the numerical value, we divide by 1m to eliminate the units. If this architecture were to use an E/E calculation mode, it would directly result in a dimensionless number. In summary, this model does not require the quantities in the numerator and denominator to be of the same dimension, which is fundamentally different from the safety factor. Moreover, the numerical values obtained from this model are much larger than the typical range of safety factor values, and these values alone do not necessarily indicate a problem; they only become meaningful when compared with the values calculated for other glacial lakes (including those that have already experienced outbursts) or with the model's own historical values. It can be said that the reciprocal calculation mode of the safety factor is a special case of the susceptibility index calculation mode, where the numerator and denominator in the susceptibility index calculation mode must be of the same dimension (E/E calculation mode, F/F calculation mode) and the numerator must not include probability terms (such as the probability of the occurrence of instantaneous triggering bodies). At the same time, the safety factor is generally defined as the ratio of the load-bearing capacity considered during the design of a structure or component to its actual load requirements under the most unfavorable load combinations, and the most significant difference between the safety factor and the glacial lake outburst susceptibility index expression is that the former does not include probability terms (such as the probability of the occurrence of instantaneous triggering bodies) and the latter is the ratio of the load side to the resistance side. The above is sufficient to explain that this model is not performing a safety factor analysis (stability analysis).

As mentioned earlier, the core of the AHP lies in its hierarchical decision-making structure, the assessment and quantification of importance, and the calculation and synthesis of weights. Even if there are variations in computational details or technical methods, as long as these core steps are retained, the method still adheres to the spirit of the AHP. The entire glacial lake outburst susceptibility index solution analysis model is also very similar in composition to the AHP. When applying the AHP to this model, the objective layer is the susceptibility of glacial lake outbursts, the criterion layer consists of the values of the three major indicators: the value of the mechanical energy contributed by the instantaneous triggering body to the dam failure location, the value of the mechanical energy contributed by the lake water to the dam failure location, and the value affecting the resistance at the dam failure location, and the alternative layer consists of the susceptibility indices of each glacial lake awaiting assessment. This shows that the hierarchical decision-making structure is satisfied. We find that the value of the mechanical energy contributed by the instantaneous triggering body to the dam failure location and the value of the mechanical energy contributed by the lake water to the dam failure location belong to the same category. They are both representative quantities of the attacking party, and their units are consistent, so they can be directly added together after being calculated separately. In this way, there are only two types of indicators: one is the representative quantity of the attacking party—potential energy, and the other is the representative quantity of the defending party—the critical failure Newton's force. After classifying the various influencing factors of the glacial lake outburst susceptibility index based on the physical process and

quantifying the three major indicators, the valuation of each indicator has been completed, which means that this model does not need to perform a consistency test. When constructing the pairwise comparison matrices for the criterion layer and the alternative layer, especially since the two types of indicators belong to different categories, the attacking party's representative quantities and the defending party's representative quantities should be separated when constructing the matrices in the criterion layer, and judgment matrices should be constructed separately. What is more special is that after the valuation of each glacial lake indicator to be assessed is solved, they all exist objectively and independently, without any subjective judgment. Therefore, each glacial lake to be assessed can have its own judgment matrix (the purpose of the judgment matrix in the criterion layer is to measure the relative importance of each indicator and assign corresponding weights), and there is no need to construct a judgment matrix followed by all glacial lake outburst susceptibility index calculations through subjective experience. In the end, the pairwise comparison matrix for the criterion layer is two 1×1 matrices. The construction of the discriminant matrix for the alternative layer is similar to the ordinary AHP. In this way, the weights for the criterion layer and the alternative layer can be determined. The two types of indicators are classified based on the physical process and follow the physical relationship (E/F calculation mode), which just serves the problem of the objective layer, so this calculation mode must be followed when calculating and synthesizing the weights. At the same time, the impact of each indicator on the glacial lake outburst susceptibility index can be comprehensively reflected in the calculation expression. It is surprising to find that the calculation results obtained by directly bringing the valuation of each indicator into the calculation mode and the results obtained through the AHP mode still have a multiple relationship, and their order remains unchanged, which is realized by assuming data.

Hypothetical Data:

Glacial Lake 1 has a mechanical energy of 8 joules (J) and a resistance of 2 newtons (N);

Glacial Lake 2 has a mechanical energy of 12 joules (J) and a resistance of 6 newtons (N);

Glacial Lake 3 has a mechanical energy of 15 joules (J) and a resistance of 3 newtons (N).

1 、Directly solve by substituting into the calculation model：

Glacial Lake 1 ： $GLOFSI = \frac{8J}{2N \times 1m} = 4$；

Glacial Lake 2 ； $GLOFSI = \frac{12J}{6N \times 1m} = 2$；

Glacial Lake 3 ： $GLOFSI = \frac{15J}{3N \times 1m} = 5$；

2、The following is the process of solving by the Analytic Hierarchy Process (AHP):

(1) Construct the pairwise comparison matrix for mechanical energy at the criterion level.

Glacial Lake 1	Mechanical Energy
Mechanical Energy	1
Glacial Lake 2	Mechanical Energy

Mechanical Energy	1
Glacial Lake 3	Mechanical Energy
Mechanical Energy	1

（2）Construct the pairwise comparison matrix for the Resistance Force at the criterion level.

Glacial Lake 1	Resistance Force
Resistance Force	1
Glacial Lake 2	Resistance Force
Resistance Force	1
Glacial Lake 3	Resistance Force
Resistance Force	1

(3) Construct the pairwise comparison matrix for mechanical energy at the alternative level.

Mechanical Energy	Glacial Lake 1	Glacial Lake 2	Glacial Lake 3
Glacial Lake 1	1	2/3	8/15
Glacial Lake 2	3/2	1	4/5
Glacial Lake 3	15/8	5/4	1

(4) Construct the pairwise comparison matrix for the Resistance Force at the alternative level.

Resistance Force	Glacial Lake 1	Glacial Lake 2	Glacial Lake 3
Glacial Lake 1	1	1/3	2/3
Glacial Lake 2	3	1	2
Glacial Lake 3	3/2	1/2	1

(4) Calculation and Synthesis of Weights

Glacial Lake 1 Mechanical Energy weight：

$$\frac{1+2/3+8/15}{(1+2/3+8/15)+(3/2+1+4/5)+(15/8+5/4+1)}=8/35$$

Glacial Lake 1 Resistance Force weight:

$$\frac{1+1/3+2/3}{(1+1/3+2/3)+(3+1+2)+(3/2+1/2+1)}=2/11$$

Integrated calculation of weights from the criterion layer and the alternative layer:

$$GLOFSI = \frac{8/35 \times 1}{2/11 \times 1} = 44/35$$

Similarly, the following can be obtained:

Glacial Lake 2： $GLOFSI = \dfrac{12/35 \times 1}{6/11 \times 1} = 22/35$

Glacial Lake 3 ； $GLOFSI = \dfrac{15/35 \times 1}{3/11 \times 1} = 55/35$

By directly substituting the indicator values into the calculation model and comparing the results with those obtained after constructing the pairwise comparison matrix through the normal Analytic Hierarchy Process (AHP), it is found that they are in a multiple relationship. The value of this multiple is always equal to the sum of the mechanical energy of each glacial lake divided by the sum of the resistance forces of the dams. At the same time, for the sake of simplicity in calculation, when calculating the pairwise comparison matrix at the criterion layer, it is usually only necessary to construct a pairwise comparison matrix for a single glacial lake.

From this, we can conclude that the model can directly omit the step of constructing the comparison matrix at the criterion and scheme layers, and there is no need to consider consistency checks. Finally, the calculation results of the glacial lakes that have already burst and those to be evaluated can be compared and sorted, and can also be compared with their own previous glacial lake susceptibility indices. After comparison with other glacial lakes to be evaluated at the same time, intervals can be divided for susceptibility assessment, and the change in the Glacial Lake Outburst Floods susceptibility index can be analyzed by comparing it with its own previous Glacial Lake Outburst Floods susceptibility index. In theory, all samples of glacial lake bursts can serve as a historical database, and the Glacial Lake Outburst Floods susceptibility assessment model will no longer be restricted by regions, further strengthening the model's universality. The above is sufficient to illustrate that it is a model that naturally embeds physical processes into the Analytic Hierarchy Process without considering subjective experience, and it is also a susceptibility assessment model.

8 Construction Method of the Analytic Hierarchy Process Based on Physical Processes

The construction of the glacial lake outburst susceptibility assessment model, from inception to result, appears very natural and completely uninvolved with subjective experience. For now, let's call the method of constructing the glacial lake outburst susceptibility assessment model based on physical processes the Analytic Hierarchy Process Based on Physical Processes (AHPBPP). The glacial lake outburst susceptibility assessment model can be considered a typical application of the AHPBPP, so it is necessary to summarize the commonalities of the AHPBPP from the application model. Therefore, I will summarize and refine the process of constructing the glacial lake outburst susceptibility assessment model to further introduce the AHPBPP.

The first step in constructing the AHPBPP is to clearly define our objective layer, and then to deeply understand the physical processes involved in the problem of the objective layer

(glacial lake outburst susceptibility). Next, classify the factors affecting the objective layer problem to establish the criterion layer (the values of mechanical energy contributed by the instantaneous triggering body to the dam failure location, the values of mechanical energy contributed by the lake water to the dam failure location, and the values affecting the resistance at the dam failure location), and clarify the physical relationships between the categorized classes that serve the objective layer problem (attacker and defender), and establish the physical relationship (the E/F calculation mode in the glacial lake outburst susceptibility model). It is essential to find the flow quantity (quantify each indicator with flow quantity, in the glacial lake outburst susceptibility model, energy is the flow quantity). Then, establish the alternative layer (susceptibility indices of each glacial lake).

Next, construct the pairwise comparison matrices for the criterion layer and the alternative layer, noting that the pairwise comparison matrices for the numerator and denominator criteria need to be constructed separately. Each entity awaiting assessment will have its own pairwise comparison matrix. Since the units of the numerator items are consistent, they can generally be combined into one item, and the same applies to the denominator items, so the pairwise comparison matrix for the criterion layer is two 1×1 matrices. This 1×1 matrix is essentially a comparison of oneself with oneself. The construction method for the pairwise comparison matrix at the alternative layer is consistent with the construction method for the pairwise comparison matrix in the standard AHP.

Finally, synthesize and calculate the weights based on the physical relationships. According to the needs of the objective layer problem, rank, zone, or compare these results with the previous susceptibility indices of the entities awaiting assessment. It is found that when the calculation results obtained by directly substituting the valuation of each indicator into the calculation mode have a multiple relationship with the results obtained by normally constructing pairwise comparison matrices (this step can be realized by assuming multiple sets of data), the model is essentially constructed. Thus, the AHPBPP is constructed.

The AHPBPP is a method for solving complex scientific problems, requiring that the problem to be solved at the objective layer can be fully summarized by indicators, and these indicators can only be divided into two categories, namely, those written in the numerator and the denominator. It is not required that the dimensions of the numerator and denominator can cancel each other out, but it is required that when all indicators are collected in the fraction, the numerical value of the fraction can exactly describe the problem to be solved, such as the susceptibility index of glacial lake outbursts.

8.1 Landslide Susceptibility Analysis Model Based on the Analytic Hierarchy Process of Physical Processes

The glacial lake outburst susceptibility model based on the Analytic Hierarchy Process of Physical Processes is a typical application of this method. Through the construction and analysis of the previous model, as well as the refinement of the method, it is not difficult to discover that the Analytic Hierarchy Process of Physical Processes, as a method, should have a certain degree of generalization capability. Therefore, this paper attempts to establish a landslide susceptibility assessment model based on the Analytic Hierarchy Process of Physical Processes.

In this context, the objective layer is landslide susceptibility; the criterion layer consists of the potential energy of the slope body at the foot of the slope, additional instantaneous

mechanical energy, and the slope's anti-sliding force; the alternative layer is the susceptibility index of each slope body. When clarifying the physical relationships among the indicators of the criterion layer, it is found that they still follow the E/F calculation mode, as shown in equation (10). Then, construct the pairwise comparison matrices for the criterion layer and the alternative layer. After merging the two parts of the numerator, the pairwise comparison matrix for the criterion layer is still a 1×1 matrix. The pairwise comparison matrix for the alternative layer is constructed according to the standard Analytic Hierarchy Process. The results obtained by directly substituting the values of the indicators into the calculation mode and those obtained through the normal Analytic Hierarchy Process still have a multiple relationship, which is realized by assuming multiple sets of data. Obtain the susceptibility indices of each landslide (including those that have already occurred), then compare these results with each other, rank them, and divide them into high and low susceptibility intervals; or compare these results with their previous indices for analysis.

$$LSI = \frac{G + \sum_{i=j=1}^{n} \psi_i E_{fj}}{F * 1m} \qquad (10)$$

In the formula, *LSI* refers to the landslide susceptibility index. We are only concerned with the specific numerical value of this ratio, so it is divided by 1 meter to eliminate the units and obtain a dimensionless number; *G* represents the potential energy of the slope body at the foot of the slope, Ψ is the probability coefficient of the instantaneous additional mechanical energy taking effect,with a maximum value of 1 and a minimum value of 0; E_f is the instantaneous additional mechanical energy; *F* is the anti-sliding force of the slope body; the unit of energy is joules, and the unit of force is newtons.

The construction of the landslide susceptibility assessment model has demonstrated that the Analytic Hierarchy Process Based on Physical Processes possesses a certain degree of generalization capability. It also builds a bridge between stability analysis and susceptibility analysis. We often believe that the more unstable landslides there are in a region, the higher the regional landslide susceptibility tends to be. This issue has been partially explained here. When the probability term coefficient Ψ is determined to be 0 or 1 and the E/E calculation mode, F/F calculation mode are employed, the more unstable the landslide, the greater the landslide susceptibility index derived from the calculation formula. If there are more unstable landslides in the region, then there are more landslides with an increased susceptibility index in the region, which implies a higher regional landslide susceptibility. Models that use E/E calculation modes, F/F calculation modes, M/M calculation modes, and similar patterns still belong to this category. The glacial lake outburst susceptibility assessment model is similar; this model will not be restricted by regional limitations.

9 The Law of Relative Quantities with Uncancelled Dimensions（量纲不相消相对量法则）

From the summary of the Analytic Hierarchy Process Based on Physical Processes and the construction of the glacial lake outburst susceptibility model and the landslide susceptibility model, we have discovered that the numerical results obtained from direct comparisons between data of different dimensions and the values obtained through constructing pairwise comparison matrices and synthesizing weights are in a multiple relationship, with this multiple being equal to the sum of the denominator values divided by the sum of the numerator values.

The process of constructing pairwise comparison matrices may seem meaningless, but it is an extremely important dimensionless process. Later, in the weight determination phase, the dimensionless weights are synthesized to ultimately obtain a dimensionless number. This dimensionless number is in a constant multiple relationship with the numerical results obtained by directly comparing data of different dimensions. This seems to be a mathematical equation, but it does not satisfy the principle of dimensional consistency. We have discovered this pattern, which can greatly simplify future calculations. It is possible to directly take the ratio of the original data with different dimensions to directly obtain the results, or to divide by the multiple relationship to get the desired values. However, the ratio of data with different dimensions results in a dimensionally consistent outcome. This can be proven under the principle of dimensional consistency, hence it is named the Law of Relative Quantities with Uncancelled Dimensions.

The Law of Relative Quantities with Uncancelled Dimensions states that when there are two or more sets of data that only have two types of dimensions, and the form of dimensions for each set of data is consistent, and the change in the ratio of the numerical values of the two data can reflect the change in some target value, the ratio of the target values of these sets of data can be solved without considering the dimensions, and the ratio of the numerical results of each set of data can be directly used as the ratio of the target values.

It may seem that this method, which directly uses the ratio of the numerical results of each set of data as the ratio of the target values without considering the dimensions, violates the principle of dimensional consistency. However, this law is deduced under the principle of dimensional consistency. Data sets with the same dimensions can not only compare themselves internally to get a dimensionless number but also compare with other sets to get a dimensionless number. Data sets with uncancelled dimensions can only compare with other data sets of the same form to get a dimensionless number. Compared with the dimensionless numbers obtained by comparing data sets with the same dimensions, the dimensionless numbers obtained by data sets with uncancelled dimensions carry less information. For example, in the field of landslide prediction analysis, if the F/F model is used and a probability term is added to the numerator, a landslide susceptibility index can be obtained. This index can be compared with the unit 1; if the landslide susceptibility index is greater than 1, the landslide is dangerous and may slide. At the same time, this index can also be compared with other indices, and the larger one is more dangerous. If the E/F model is used in the field of landslide prediction analysis, the size of the dimensionless number itself is meaningless, and it only becomes meaningful when compared with the dimensionless numbers of other groups. If a new set of data with uncancelled dimensions is added, to allow the new set of data with uncancelled dimensions to participate in the comparison, all previous comparisons must be re-done. At this time, it is necessary to reconstruct the pairwise comparison matrices and re-synthesize the weights, which makes the calculation somewhat complex. If the Law of Relative Quantities with Uncancelled Dimensions is used, the results can be directly obtained, eliminating the complex calculation process.

9.1 Proof of the Law of Relative Quantities with Uncancelled Dimensions

Assume there are multiple sets of data E_1/F_1, E_2/F_2, E_3/F_3, E_4/F_4, ..., where the units of E are consistent and the units of F are consistent. It is stipulated that E and F can both represent a

pair of opposing quantities, such as the destructive force and resistance in a landslide. For instance, E represents energy, and F represents Newtonian force. The susceptibility index of a landslide is jointly controlled by the destructive force and resistance; the actual values of the destructive force and resistance govern the magnitude and variation of the landslide susceptibility index. When all sets of data are in this form, the relative size of the susceptibility index is independent of the units. Susceptibility is a relative concept, indicating the relative likelihood of an individual unit exhibiting certain signs within a group. When using E to represent the destructive force and F to represent the resistance, one must clarify the relative magnitude of the susceptibility index of each individual within a group. At this point, each E is compared with others to obtain the relative value of E within the group, which is a dimensionless number; each F is compared with others to obtain the relative value of F within the group. By comparing the relative value of an individual E within the group with the relative value of F, we obtain the relative susceptibility index of the individual within the group. The ratio of the relative susceptibility indices of each individual is the final susceptibility, which can then be ranked. The relative susceptibility index of an individual will change with the increase or decrease of individuals within the group. This change can cause many computational troubles in solving some practical problems, and errors are prone when the amount of data increases. The Law of Relative Quantities with Uncancelled Dimensions effectively addresses this issue. Here, since there is only one numerator and one denominator, the weights of the numerator and denominator when compared to themselves are both 1. This can also be disregarded in the future.

(1) Construct the pairwise comparison matrix for the numerator E.

numerator E	E_1	E_2	E_3	E_4
E_1	1	E_1/E_2	E_1/E_3	E_1/E_4
E_2	E_2/E_1	1	E_2/E_3	E_2/E_4
E_3	E_3/E_1	E_3/E_2	1	E_3/E_4
E_4	E_4/E_1	E_4/E_2	E_4/E_3	1

(2) Construct the pairwise comparison matrix for the denominator F.

分母 F	F_1	F_2	F_3	F_4
F_1	1	F_1/F_2	F_1/F_3	F_1/F_4
F_2	F_2/F_1	1	F_2/F_3	F_2/F_4
F_3	F_3/F_1	F_3/F_2	1	F_3/F_4
F_4	F_4/F_1	F_4/F_2	F_4/F_3	1

(3) Calculation and synthesis of weights.

The relative weight of E_1:

$$\frac{1+E_1/E_2+E_1/E_3+E_1/E_4}{(1+E_1/E_2+E_1/E_3+E_1/E_4)+(1+E_2/E_1+E_2/E_3+E_2/E_4)+(1+E_3/E_1+E_3/E_2+E_3/E_4)+(1+E_4/E_1+E_4/E_2+E_4/E_3)}$$

$$=\frac{E_1}{E_1+E_2+E_3+E_4}$$

The relative weight of E_2: $\frac{E_2}{E_1+E_2+E_3+E_4}$

The relative weight of E_3: $\frac{E_3}{E_1+E_2+E_3+E_4}$

The relative weight of E_4: $\frac{E_4}{E_1+E_2+E_3+E_4}$

The relative weight of F_1: $\frac{F_1}{F_1+F_2+F_3+F_4}$

The relative weight of F_2: $\frac{F_2}{F_1+F_2+F_3+F_4}$

The relative weight of F_3: $\frac{F_3}{F_1+F_2+F_3+F_4}$

The relative weight of F_4: $\frac{F_4}{F_1+F_2+F_3+F_4}$

The relative susceptibility index of E_1/F_1: $\frac{(F_1+F_2+F_3+F_4)\ E_1}{(E_1+E_2+E_3+E_4)\ F_1}$

The relative susceptibility index of E_2/F_2: $\frac{(F_1+F_2+F_3+F_4)\ E_2}{(E_1+E_2+E_3+E_4)\ F_2}$

The relative susceptibility index of E_3/F_3: $\frac{(F_1+F_2+F_3+F_4)\ E_3}{(E_1+E_2+E_3+E_4)\ F_3}$

The relative susceptibility index of E_4/F_4: $\frac{(F_1+F_2+F_3+F_4)\ E_4}{(E_1+E_2+E_3+E_4)\ F_4}$

It is not difficult to discover that, when considering only the numerical values without regard to the units, the value of E/F is $\sum_{i=1}^{n} E_i / \sum_{i=1}^{n} F_i$ times the relative susceptibility index of E/F.

If we let the value that considers only the numerical value without the unit be A/B, and the corresponding relative susceptibility index be a/b, with a total of n sets of data, then we have:

$$\frac{A}{B}=\frac{a\sum_{i=1}^{n}A_i}{b\sum_{i=1}^{n}B_i} \tag{11}$$

The Law of Relative Quantities with Uncancelled Dimensions is proven. Of course, relative quantities with cancelled dimensions also satisfy this theorem, but generally, when dimensions are cancelled, a dimensionless number can be directly obtained without the need to construct a pairwise comparison matrix. Relative quantities with cancelled dimensions are more commonly encountered in our practical applications, while relative quantities with uncancelled dimensions are rarely encountered.

In summary, the Law of Relative Quantities with Uncancelled Dimensions brings the following benefits: First, it provides a theoretical basis for models such as the E/F model in susceptibility issues, making seemingly impossible models possible and inspiring similar problems in other fields; second, it greatly simplifies calculations. For example, without using the Law of Relative Quantities with Uncancelled Dimensions, if 100 landslide susceptibility indices have been calculated, there is a significant amount of computation involved, including calculating the relative weights of the numerator and the denominator separately, and then synthesizing them. After that, the ratios of the 100 landslide susceptibility indices are obtained. If the Law of Relative Quantities with Uncancelled Dimensions is adopted, the calculation can directly ignore the dimensions and only consider the numerical values. Then, the ratios can be obtained and sorted. There is also a worse-case scenario: if there are 101 landslides in total and one is missed. The susceptibility indices of the original 100 landslides calculated without using the Law of Relative Quantities with Uncancelled Dimensions would have to be recalculated due to the omission of one landslide, increasing the computational load. If the Law of Relative Quantities with Uncancelled Dimensions was adopted from the beginning, the results could be directly compared with the previous 100 landslides without considering dimensions, only considering numerical values. Third, the numerical values obtained by directly ignoring dimensions have the same functionality as those obtained by considering dimensions, and are even more convenient than considering dimensions. The Law of Relative Quantities with Uncancelled Dimensions broadens the scope of data comparison in problems where it is applicable.

10 Conclusion

This paper has proposed a theoretical model for the assessment of glacial lake outburst susceptibility, further introduced the Analytic Hierarchy Process Based on Physical Processes, and applied this method in the construction of a landslide susceptibility assessment model, and further proposed the Law of Relative Quantities with Uncancelled Dimensions.

While the model is novel, it inherently possesses complexity, especially in accurately obtaining its numerous parameters. The pursuit of precise parameter estimation is not just a scientific endeavor but a crucial step in enhancing the model's predictive power and practical application value, representing an important area for the future development of

this model. The construction of the model is similar in architecture to safety factor assessment, aiming to reflect the comparison between attacking forces (such as triggering events and the mechanical energy of lake water) and defensive forces (resistance offered by the dam body). This comparison is not just a theoretical construct but the key to systematic assessment and precise approximation of the susceptibility index. The model clarifies the relationship between susceptibility and stability.

However, the development of the model is not without challenges. The difficulty in accurately determining certain parameters, such as the exact location of the dam's failure core, demonstrates the necessity for continuous improvement. Nonetheless, the application of the model is not limited by region, and it involves no subjective experience from beginning to end, emphasizing its robustness and universal applicability. The Analytic Hierarchy Process Based on Physical Processes is expected to be applied in other fields in the future. The Law of Relative Quantities with Uncancelled Dimensions broadens the scope of data comparison and greatly simplifies calculations in solving some practical problems.

Note: The model is in its initial stage and is ongoing work that requires rigorous validation and iterative enhancement. We extend an open invitation to scholars and practitioners to contribute their insights and suggestions, with the shared goal of improving the model's accuracy and broadening its practical relevance.

ACKNOWLEDGEMENTS

There are no conflicts of interest, and the views expressed represent my personal opinion only.Like-minded friends are welcome to email me.

References

[1]Allen, S. K., Rastner, P., Arora, M., Huggel, C. & Stofel, M. Lake outburst and debris flow disaster at Kedarnath, June 2013: hydrometeorological triggering and topographic predisposition[J]. Landslides 13, 1479–1491 (2016). https://doi.org/10.1007/s10346-015-0584-3

[2]Bolch, T., Peters, J., Yegorov, A., et al. Identification of potentially dangerous glacial lakes in the northern Tien Shan[J]. Natural Hazards, 2011, 59: 1691-1714 https://doi.org/10.1007/s11069-011-9860-2

[3]Carrivick, J. L. et al. Mass loss of glaciers and ice caps across Greenland since the Little Ice Age[J]. Geophys. Res. Lett. 50, e2023GL103950 (2023). https://doi.org/10.1029/2023gl103950

[4]Cui, P., Chen, R., Xiang, L. Z., et al. (2003). Formation, evolution, and mitigation strategies of glacial lake outburst debris flow. Quaternary Sciences, (6), 621-628. https://doi.org/10.3969/j.issn.1001-5641.2003.06.018

[5]Carey, M. Living and dying with glaciers: people's historical vulnerability to avalanches and outburst floods in Peru. Glob. Planet. Change 47, 122–134 (2005). https://doi.org/10.1016/j.gloplacha.2004.10.007

[6]Carrivick, J. L., Tweed, F. S. Proglacial lakes: character, behaviour and geological importance[J]. Quaternary Science Reviews, 2013, 78: 34-52.https://doi.org/10.1016/j.quascirev.2013.07.028
[7]Clague J.J., Evans S.G. A review of catastrophic drainage of moraine-dammed lakes in British Columdia. Quaternary Science Reviews, 2000, 19:1763-1783. https://doi.org/10.1016/s0277-3791(00)00090-1
[8]Costa, J. E., Schuster, R. L. The formation and failure of natural dams[J]. Geological society of America bulletin, 1988, 100(7): 1054-1068 .https://doi.org/10.1130/0016-7606(1988)100%3C1054:tfafon%3E2.3.co;2
[9]Carrivick JL, Tweed FS. 2016. A global assessment of the societal impacts of glacier outburst floods (in English). Global Planet Change. 144:1–16.https://doi.org/10.1016/j.gloplacha.2016.07.001
[10]Cui, P., Guo, X., Jiang, T., Zhang, G., & Jin, W. (2019). Disaster effect induced by Asian Water Tower change and mitigation strategies. Bulletin of the Chinese Academy of Sciences, 32(9), 985-992. https://doi.org/10.16418/j.issn.1000-3045.2019.11.014
[11]Dahong Zhang,Gang Zhou,Wen Li,Lei Han,Shiqiang Zhang,Xiaojun Yao, & Hongyu Duan (2023). A robust glacial lake outburst susceptibility assessment approach validated by GLOF event in 2020 in the Nidu Zangbo Basin, Tibetan Plateau. CATENA, 220 (0), 106734-106734. https://doi.org/10.1016/j.catena.2022.106734
[12]Diamantidis, D., Tanner, P., Holicky, M., Madsen, H. O., & Sykora, M. (2024). On reliability assessment of existing structures. Structural Safety. https://doi.org/10.1016/j.strusafe.2024.102452
[13]Dubois, D., & Prade, H. (1980). "Fuzzy Sets and Systems: Theory and Applications." Academic Press.
[14]Emmer, A. (2013). The causes and mechanisms of moraine-dammed lake failures in the Cordillera Blanca, North American Cordillera, and Himalayas. AUC Geographica, 48(2), 5-15. https://doi.org/10.14712/23361980.2014.23
[15]Emmer, A. (2017). Geomorphologically effective floods from moraine-dammed lakes in the Cordillera Blanca, Peru. Quaternary Science Reviews, 177, 220-234.https://doi.org/10.1016/j.quascirev.2017.10.028
[16]Emmer A, Vil ımek V, Zapata ML. 2018. Hazard mitigation of glacial lake outburst floods in the Cordillera Blanca (Peru): the effectiveness of remedial works. J Flood Risk Manage. 11: S489–S501.https://doi.org/10.1111/jfr3.12241
[17]Gardelle, J., Arnaud, Y., Berthier, E. Contrasted evolution of glacial lakes along the Hindu Kush Himalaya mountain range between 1990 and 2009[J]. Global and Planetary Change, 2011, 75(1- 2): 47-55 https://doi.org/10.1111/jfr3.12241
[18]Harrison, S., Kargel, J. S., Huggel, C., et al. (2018). Climate change and the global pattern of moraine-dammed glacial lake outburst floods. Cryosphere, 12(4), 1-28.https://doi.org/10.5194/tc-12-1195-2018
[19]Haeberli, W., Schaub, Y., & Huggel, C. (2017). Increasing risks related to landslides from degrading permafrost into new lakes in de-glaciating mountain ranges. In T. E. C. O. P. E. I.-P.-A. R. CONFERENCE (Ed.), Geomorphology; Evora,

PORTUGAL (pp. 405-417).https://doi.org/10.1016/j.geomorph.2016.02.009
[20]Hegglin, E., Huggel, C. An integrated assessment of vulnerability to glacial hazards[J]. Mountain Research and Development, 2008, 28(3/4): 299-309.https://doi.org/10.1659/mrd.0976
[21]Huggel, C., Haeberli, W., Kääb, A., et al. An assessment procedure for glacial hazards in the Swiss Alps[J]. Canadian Geotechnical Journal, 2004, 41(6): 1068-1083.https://doi.org/10.1659/mrd.0976
[22]Harrison S, Kargel JS, Huggel C, Reynolds J, Shugar DH, Betts RA, Emmer A, Glasser N,Haritashya UK, Klimes J, et al. 2018. Climate change and the global pattern of moraine-dammed glacial lake outburst floods. Cryosphere. 12(4):1195–1209.https://doi.org/10.1139/t04-053
[23]IPCC: Climate change 2013– the physical science basis, in: Con tribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, edited by: Stocker, T. F., Qin, D., Plattner, G.-K., Tignor, M., Allen, S. K., Boschung, J., Nauels, A., Xia, Y., Bex, V., and Midgley, P. M., CambridgeUniversity Press, Cambridge, UK and NewYork,NY, USA, 2013.
[24]International Organization for Standardization. (2017). ISO 13822:2017 - Basis for design of structures - Assessment of existing structures. https://www.iso.org/standard/63485.html
[25]International Code Council. (2018). International Building Code. https://www.iccsafe.org/codes/international-building-code/
[26]Jiajia Gao,Jun Du, & Zhuoma Yixi (2023). Forewarning Model for Glacial Lake Outburst in Southeast Tibet. Applied sciences, 13 (3), 1797-1797. https://doi.org/10.3390/app13031797 36
[27]Lee, E. et al. Accelerated mass loss of Himalayan glaciers since the Little Ice Age. Sci. Rep. 11, 24284 (2021).https://doi.org/10.1038/s41598-021-03805-8
[28]Liu, J.F.; Cheng, Z.L.; Chen, X.Q. The hazard assessment of glacier-lake outbust in Palongzangbu river from Ranwu to Peilong.J. Mt. Sci. 2012, 30, 369–377. 10.16089/j.cnki.1008-2786.2012.03.017
[29]Liu Jing-Jing, Cheng Zun-Lan, Su Peng-Cheng. The relationship between temperature fluctuation and Glacial Lake Outburst Floods in Tibet,China.QUATERNARY INTERNATIONAL[J]. 2014, 321: 78-87
https://doi.org/10.1016/j.quaint.2013.11.023
[30]Liu, J., Tang, C., & Cheng, Z. (2013). The Two Main Mechanisms of Glacier Lake Outburst Flood in Tibet, China. Journal of Mountain Science, 10(2), 239–248. https://doi.org/10.1007/s11629-013-2517-8
[31]Liu, M., Chen, N., Zhang, Y., and Deng, M. (2020). Glacial lake inventory and lake outburst flood/debris flow hazard assessment after the gorkha earthquake in the bhote Koshi basin. Water. 12, 464. doi:10.3390/w12020464.
[32]Liu, J.-J., Cheng, Z.-L., and Li, Y. (2014). The 1988 glacial lake outburst flood in Guangxieco Lake, Tibet, China. Nat. Hazards Earth Syst. Sci. 14, 3065. doi:10.5194/nhess-14-3065-2014
[33]Marta, S. et al. The retreat of mountain glaciers since the Little Ice Age: a spatially explicit database. Data 6, 107 (2021).https://doi.org/10.3390/data6100107

[34]Mckillop R J, Clague J J. Statistical, remote sensing-based approach for estimating the probability of catastrophic drainage from moraine-dammed lakes in Southwestern British Columbia[J]. Global & Planetary Change, 2007, 56(1-2):153-171.https://doi.org/10.1016/j.gloplacha.2006.07.004
[35]Mergili, M., Schneider, J. Regional-scale analysis of lake outburst hazards in the southwestern Pamir, Tajikistan, based on remote sensing and GIS[J]. Natural Hazards and Earth System Sciences, 2011, 11(5): 1447-1462.
https://doi.org/10.5194/nhess-11-1447-2011
[36]Nie, Y., Deng, Q., Pritchard, H. D., Carrivick, J. L., Ahmed, F., Huggel, C., Liu, L., Wang, W., Lesi, M., Wang, J., Zhang, H., Zhang, B., Lü, Q., & Zhang, Y. (2023). Glacial lake outburst floods threaten Asia's infrastructure.Science Bulletin, 68(7), 1361–1365. https://doi.org/10.1016/j.scib.2023.05.035
[37]Nie, Y., Liu, Q., Wang, J.D., et al. (2018). An inventory of historical glacial lake outburst floods in the Himalayas based on remote sensing observations and geomorphological analysis. Geomorphology, 308, 91-106. https://doi.org/10.1016/j.geomorph.2018.02.002
[38]Nitesh Khadka,Xiaoqing Chen,Yong Nie,Sudeep Thakuri,Guoxiong Zheng, & Guoqing Zhang (2021). Evaluation of Glacial Lake Outburst Flood Susceptibility Using Multi-Criteria Assessment Framework in Mahalangur Himalaya. Frontiers in Earth Science, 8 (0), 0-0. https://doi.org/10.3389/feart.2020.601288
[39]Otto, J.-C. (2019). "Proglacial lakes in high mountain environments," in Geomorphology of proglacial systems. New York: Springer.
[40]Qin D, Ding Y, Xiao C, . Cryospheric Science: research framework and disciplinary system[J]. National Science Review, 2018, 5(2): 255-268.https://doi.org/10.1093/nsr/nwx108
[41]Richardson, S. D., & Reynolds, J. M. (2000). An overview of glacial hazards in the Himalayas. Quaternary International, 65–66, 31-47. https://doi.org/10.1016/S1040-6182(99)00033-7
[42]Reynolds, J. M. Development of glacial hazard and risk minimization protocols in rural environments: Guidelines for the management of glacial hazards and risks[M]. Reynolds Geo-Sciences Independent Geological Consultants, Report No: R7816, 2003.
[43]Robin Neupane, Huayong Chen & Chunran Cao (2019) Review of moraine dam failure mechanism, Geomatics, Natural Hazards and Risk, 10:1, 1948-1966, DOI:10.1080/19475705.2019.1652210
[44]R. Worni, C. Huggel, J.J. Clague, Y. Schaub, M. Stoffel Coupling glacial lake impact, dam breach, and flood processes: a modeling perspective Geomorphology, 224 (2014), pp. 161-176, 10.1016/j.geomorph.2014.06.031
[45]Shugar, D. H., Burr, A., Haritashya, U. K., et al. Rapid worldwide growth of glacial lakes since 1990[J]. Nature Climate Change, 2020, 10(10): 939-945.https://doi.org/10.1038/s41558-020-00896-9
[46]Sattar, A., Haritashya, U.K., Kargel, J.S., et al. (2022). Transition of a small Himalayan glacier lake outburst flood to a giant transborder flood and debris flow. Scientific Reports, 12(1), 12421. https://doi.org/10.1038/s41598-02216337-6
[47]Steijn, H. V. Debris-flow magnitude-frequency relationships for mountainous

regions of Central and Northwest Europe[J]. Geomorphology, 1996, 15(3): 259-273.https://doi.org/10.1016/0169-555X(95)00105-X
[48]Somos-Valenzuela MA, Chisolm RE, Rivas DS, Portocarrero C, McKinney DC. 2016.Modeling a glacial lake outburst flood process chain: the case of Lake Palcacocha andHuaraz, Peru. Hydrol Earth Syst Sci. 20(6):2519–2543.https://doi.org/10.5194/hess-20-2519-2016
[49]Song, E.-X., Kong, Y.-F., & Yang, J. (2016). Discussion of safety factor definitions and computation methods for geotechnical structures. Engineering Mechanics, 33(11), 1-10.https://doi.org/10.1061/(ASCE)EM.1943-7889.0001010
[50]Saaty, T. L. (1980). The Analytic Hierarchy Process. New York, NY: McGraw-Hill.
[51]Saaty, T. L. (1990). How to Make a Decision: The Analytic Hierarchy Process. European Journal of Operational Research, 48(1), 9-26.10.1016/0377-2217(90)90146-X
[52]Saaty, T. L. (1994). Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process. Pittsburgh, PA: RWS Publications.
[53]Saaty, T. L. (1996). Multicriteria Decision Making: The Analytic Hierarchy Process. International Journal of Services Sciences, 1(1), 33-45.
[54]Saaty, T. L., & Vargas, L. G. (2001). The Analytic Network Process: Decision Making with Dependence and Feedback (2nd ed.). Pittsburgh, PA: RWS Publications.
[55]Saaty, T. L. (2005). Theory and Applications of the Analytic Network Process: Decision Making with Benefits, Opportunities, Costs, and Risks. Pittsburgh, PA: RWS Publications
[56]Saaty, T. L. (2008). Relative Measurement and Its Generalization in Decision Making: Why Ordered Sets? Mathematical and Computer Modelling, 48(3-4), 392-405.10.1016/j.mcm.2006.12.004
[57]Tzeng, G. H., & Huang, J. J. (2011). "Multiple Attribute Decision Making: Methods and Applications." CRC Press.10.1201/b11032
[58]Veh, G., Korup, O., von Specht, S., Roessner, S., and Walz, A. (2019). Unchanged frequencyofmoraine-dammedglaciallakeoutburstfloodsintheHimalaya. Nat. Clim. Change. 9, 379. doi:10.1038/s41558-019-0437-5
[59]Wang, W., Yao, T., Gao, Y., Yang, X., & Kattel, D. B. (2011). A First-order Method to Identify Potentially Dangerous Glacial Lakes in a Region of the Southeastern Tibetan Plateau. Mountain Research and Development, 31(2), 122-130. http://dx.doi.org/10.1659/MRD-JOURNAL-D-10-00059.1
[60]Wang, X.; Liu, S.Y. An Overview of researches on moraine-dammed lake outburst flood hazards. J. Glaciol. Geocryol. 2007, 29,626–635.https://doi.org/10.1016/j.jgg.2007.08.006
[61]Xu, D. (1988). Characteristics of debris flow caused by outburst of glacial lake in Boqu river, Xizang, China, 1981. Geojournal, 17(4), 569-580. https://doi.org/10.1007/BF00209443.
[62]Zhou, B., Zou, Q., Jiang, H., Yang, T., Zhou, W.-T., Chen, S.-Y., Yao, H.-K. (2023). Process-driven susceptibility assessment of glacial lake outburst debris flow in the Himalayas under climate change. Advances in Climate Change Research.

https://doi.org/10.1016/j.accre.2023.11.002 26.
[63]Zhang, G., Carrivick, J. L., Emmer, A., Shugar, D. H., Veh, G., Wang, X., Labedz, C., Mergili, M., Huss, M., Allen, S., & Lützow, N. (2024). Characteristics and changes of glacial lakes and outburst floods. Nature Reviews Earth & Environment. https://doi.org/10.1038/s43017-024-00554-w
[64]Zhang, D., Zhou, G., Li, W., Zhang, S., Yao, X., & Duan, H. (2023). A robust glacial lake outburst susceptibility assessment approach validated by GLOF event in 2020 in the Nidu Zangbo Basin, Tibetan Plateau. Catena, 220, 106734. https://doi.org/10.1016/j.catena.2022.106734
[65]Zhang, T., Wang, W., Gao, T., An, B., & Yao, T. (2022). An integrative method for identifying potentially dangerous glacial lakes in the Himalayas. Science of the Total Environment, 806, 150442. https://doi.org/10.1016/j.scitotenv.2021.150442
[66]Zadeh, L. A. (1965). "Fuzzy sets." Information and Control, 8(3), 338-353. 10.1016/S0019-9958(65)90241X